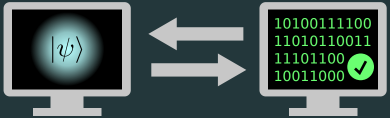
Classical verification of quantum computation



Greg Kahanamoku-Meyer

May 3, 2022

Focus of today

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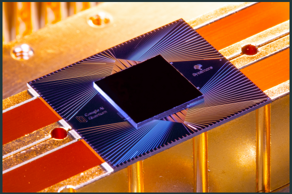
- "Verifier" + communication is entirely classical
- No assumptions about how prover works

Quantum computational advantage

Experiments claiming that their output can’t be simulated classically:



Random circuit sampling
[Google, 2019]

Gaussian boson sampling
[USTC, 2020]

...

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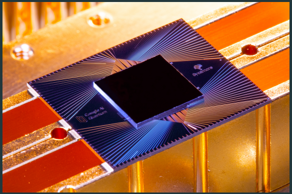

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• • •

- How hard is it *really* to classically simulate?
- If indeed we can't simulate, how do we check that it's *correct*?

How hard is it to classically simulate?

Focusing on Google's random circuit sampling experiment with 53 qubits:

Complexity theory suggests it's hard.

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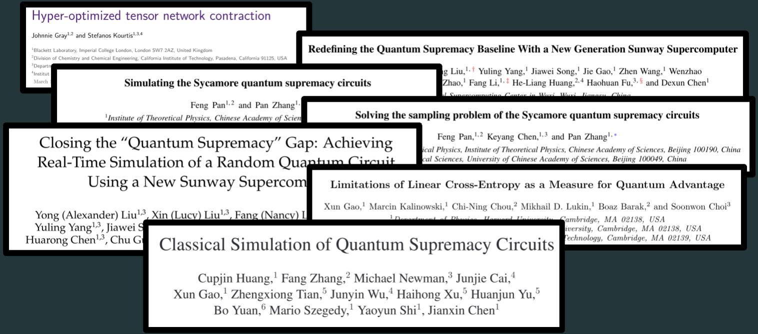

What does it mean for a computation to be classically hard?

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Complexity theory

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Takeaway: Complexity theory tells us how the hardness of a problem *scales*, but not the actual cost for specific instances.

Best strategy for finding cost in practice: **have a bunch of people try it.**

Quantum computational advantage

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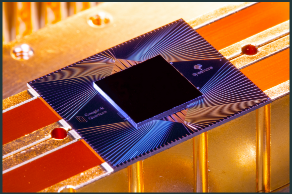

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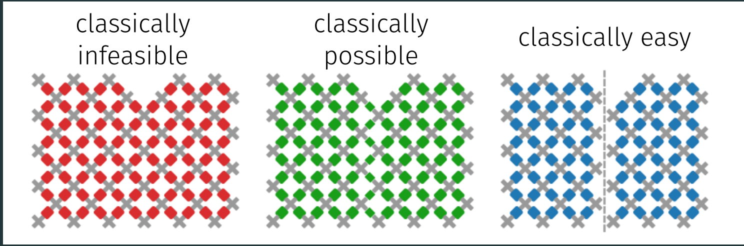

Idea: **extrapolate** correctness from simpler circuits.

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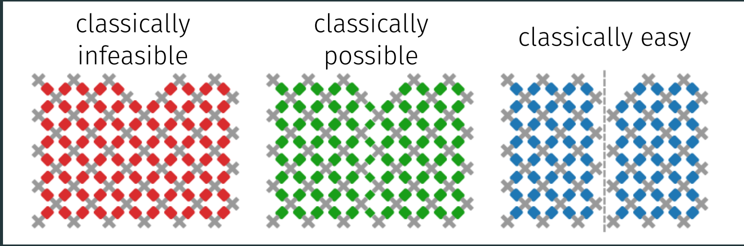

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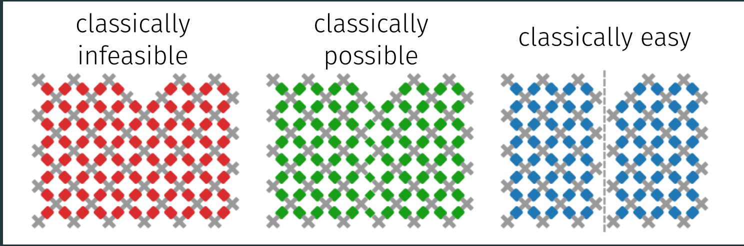

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Ideally:

- Remove need for extrapolations/assumptions in verification process
- Not need a supercomputer to do it

Robust, verifiable quantum computational advantage

We want a test with three properties:

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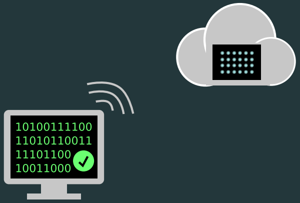

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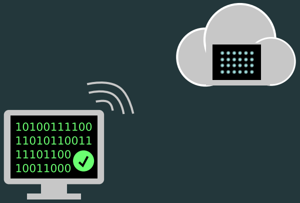

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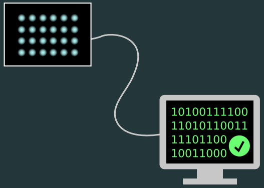

Local: robust demonstration of the power of quantum computation

"Qubits prove their power to humanity"

Connection to cryptography

> "Making sure certain things computationally hard, while keeping others easy."

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Our goal: a **"cryptographic proof of quantumness"**

Near-term verifiable quantum advantage

Trivial solution: Shor's algorithm

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Trivial solution: Shor's algorithm... but we want to do near-term!

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NISQ: Noisy Intermediate-Scale Quantum devices

Sampling problems
e.g. random circuits, Boson sampling, ...
✓ NISQ feasible
✗ Efficiently verifiable

Number theory problems
e.g. factoring, discrete logarithm, ...
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add structure

make less costly

???
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Adding structure to sampling problems

Generically: seems hard.

The point of random circuits is that they don't have structure!

IQP

Example: sampling "IQP" circuits (products of Pauli X's)

$$H = X_0 X_1 X_3 + X_1 X_2 X_4 X_5 + \cdots \tag{1}$$

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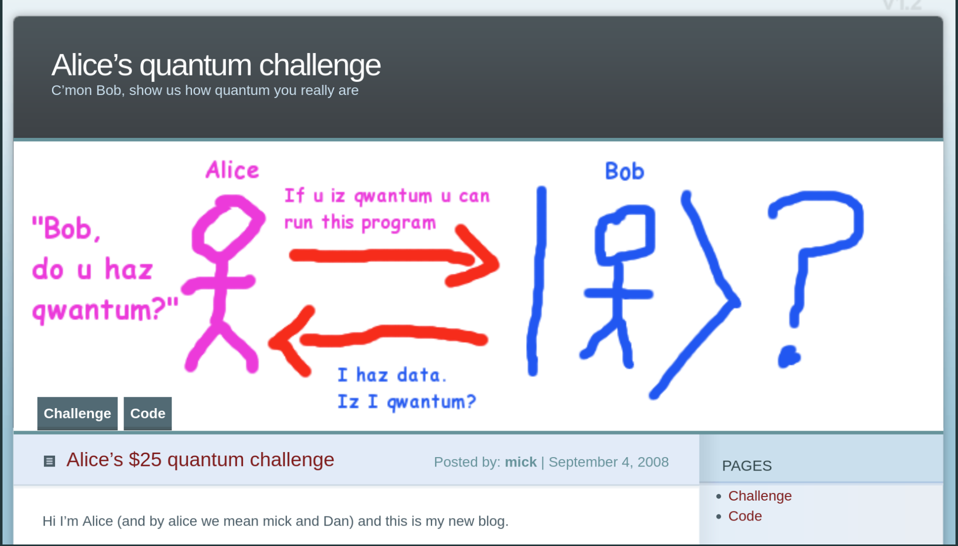
  - Is it possible to simulate this class of circuits?
  - Is there some way to pass the test *without* simulating the circuit?

The $25 challenge



IQP: is it possible to simulate classically?

Classical simulation of commuting quantum computations implies collapse of the polynomial hierarchy

By Michael J. Bremner[1,*], Richard Jozsa[2] and Dan J. Shepherd[3]

[1] *Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstrasse 2, Hannover 30167, Germany*
[2] *DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK*
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PRL **117,** 080501 (2016) PHYSICAL REVIEW LETTERS week ending 19 AUGUST 2016

Average-Case Complexity Versus Approximate Simulation of Commuting Quantum Computations

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… and in practice, it seems to be infeasible for > 50 qubits…

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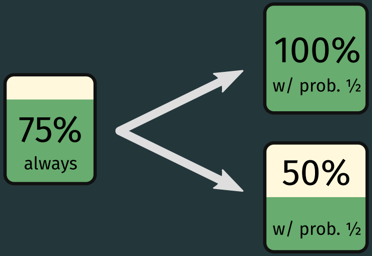

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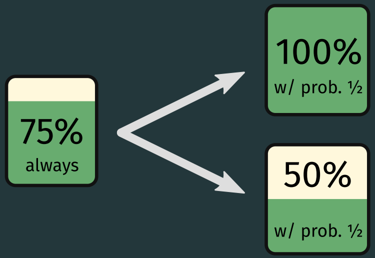

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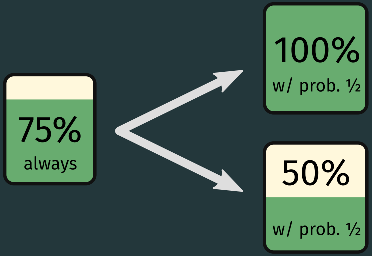

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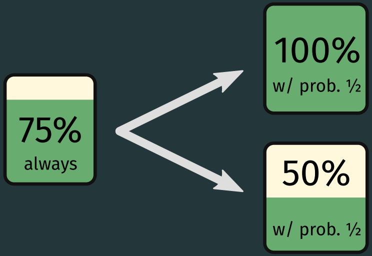

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With knowledge of $\vec{s}$, trivial to classically pass test.

Breaking the IQP protocol

Trying it against their verification code...

```
$ ./IQPwn challenge.dat
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$ ./IQPwn challenge.dat
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Congratulations; you have what appears to be a
working quantum computer!
Dataset accepted as proof!
$ 
```

Near-term verifiable quantum advantage

NISQ: Noisy Intermediate-Scale Quantum devices

Sampling problems
e.g. random circuits, Boson sampling, ...
- ✓ NISQ feasible
- ✗ Efficiently verifiable

Number theory problems
e.g. factoring, discrete logarithm, ...
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add structure

make less costly

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Making number theoretic problems less costly

Fully solving a problem like factoring is “overkill”

Making number theoretic problems less costly

Fully solving a problem like factoring is “overkill”

Can we demonstrate quantum *capability* without needing to solve such a hard problem?

Zero-knowledge proofs: differentiating colors

You are red/green colorblind, your friend is not.

How can they use a red ball and green ball to convince you that they see color?

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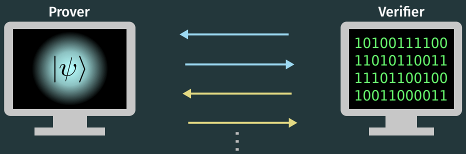
You (color blind) $\Leftrightarrow$ Classical verifier
Seeing color $\Leftrightarrow$ Quantum capability

Goal: find protocol **as verifiable and classically hard as factoring**—
but **less expensive than actually finding factors (via Shor)**

Interactive proofs of quantumness

Multiple rounds of interaction between the prover and verifier



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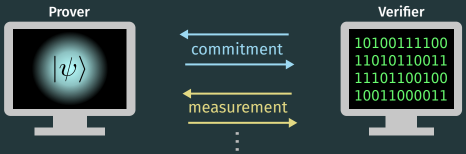

Round 1: Prover commits to holding a specific quantum state

Round 2: Verifier asks for measurement in specific basis, prover performs it

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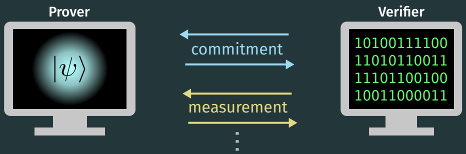

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By randomizing choice of basis and repeating interaction,
can ensure prover would respond correctly in *any* basis

State commitment (round 1): trapdoor claw-free functions

How does the prover commit to a state?

Consider a 2-to-1 function f:
for all y in range of f, there exist (x_0, x_1) such that $y = f(x_0) = f(x_1)$.

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Generating a valid state without trapdoor uses superposition + wavefunction collapse—inherently quantum!

Trapdoor claw-free function example

$$f(x) = x^2 \bmod N, \text{ where } N = pq$$

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Example: $4^2 \equiv 11^2 \equiv 16 \pmod{35}$; and $11 - 4 = 7$

Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18

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arXiv:1804.00640

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Protocol requires strong claw-free property:

For any x_0, hard to find even **a single bit** about x_1.

arXiv:1804.00640

Trapdoor claw-free functions

Function family	Trapdoor	Claw-free	Strong claw-free
Learning-with-Errors [1]	✓	✓	✓
Ring Learning-with-Errors [2]	✓	✓	✗
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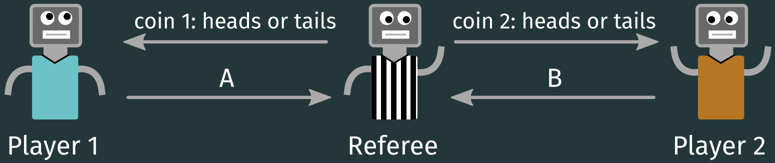
Aside: the CHSH game (Bell test)

Cooperative two-player game; players can't communicate (non-local).



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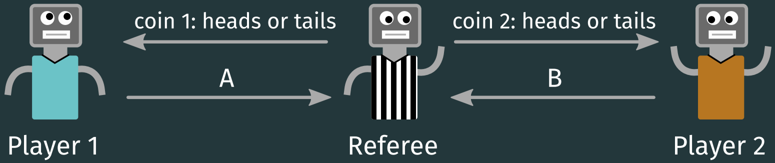

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Classical optimal strategy: return equal values, hope you didn't both get heads. 75% success rate.

Can we do better with entanglement?

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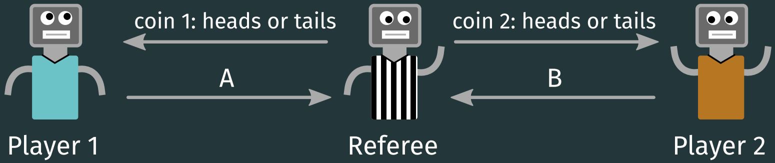

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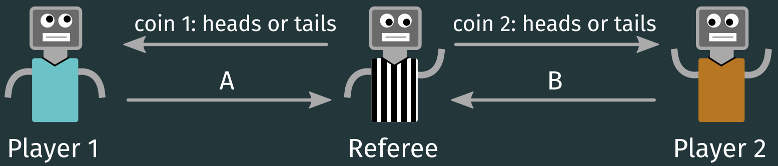

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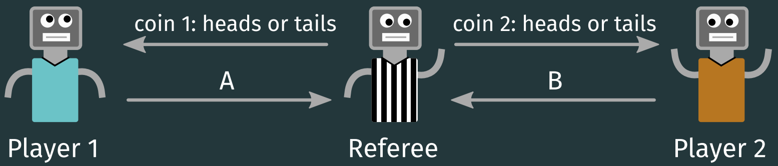

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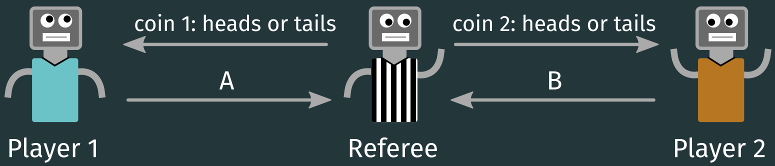

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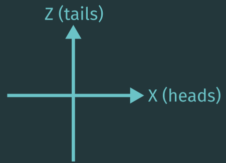

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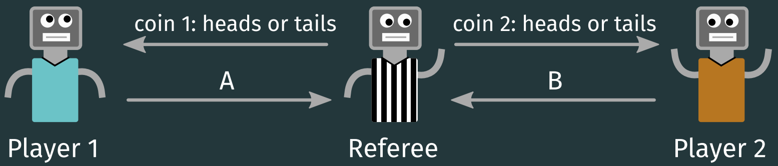

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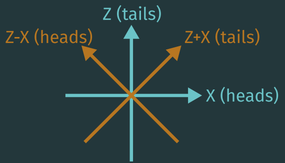

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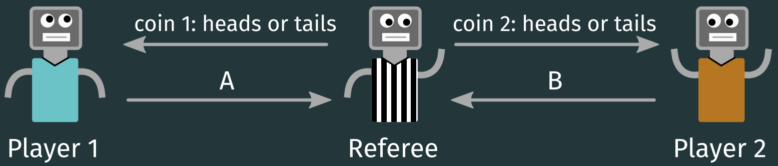

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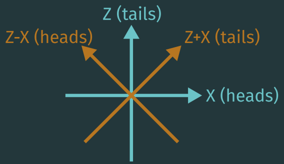

Quantum: $\cos^2(\pi/8) \approx 85\%$

Classical: 75%

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Replace X basis measurement with "single-qubit CHSH game"

Interactive measurement: computational Bell test

Two-step process: "condense" x_0, x_1 into a single qubit, and then do a "Bell test."

$\vdots$ $\qquad\qquad$ $\vdots$ $\qquad\qquad$ $\vdots$

$|x_0\rangle |x_0 \cdot r\rangle + |x_1\rangle |x_1 \cdot r\rangle$ $\xleftarrow{\quad r \quad}$ Pick random bitstring r

Measure all but ancilla in X basis $\xrightarrow{\quad d \quad}$

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

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Cryptographic secret (here) $\Leftrightarrow$ Non-communication (Bell test)

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This protocol can use any trapdoor claw-free function!

GDKM, Choi, Vazirani, Yao ’21 (arXiv:2104.00687)

Computational Bell test: classical bound

Run protocol many times, collect statistics.

p_Z: Success rate for Z basis measurement.

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Note: Let $p_Z = 1$. Then for p_{Bell}:

Classical bound 75%, ideal quantum $\sim 85\%$. Same as regular Bell test!

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- **Result:** new protocol that allows proof of quantumness using any trapdoor claw-free function, including $x^2 \bmod N$

Asymptotically: evaluating $x^2 \bmod N$ requires $\mathcal{O}(n \log n)$ gates; $a^x \bmod N$ in Shor requires $\mathcal{O}(n^2 \log n)$

(can also use other TCFs, and other optimizations...)

Moving towards efficiently-verifiable quantum advantage in the near term

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Circuit sizes

- Removing need for strong claw-free property allows use of “easier” functions
- Measurement-based uncomputation scheme [2]

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Principle of delayed measurement: delaying all measurements to the end of a circuit doesn't affect the measurement statistics.

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Q: Why is mid-circuit measurement necessary for these protocols?

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Q: Why is mid-circuit measurement necessary for these protocols?

Other applications of mid-circuit measurement:

- Quantum error correction
- Quantum machine learning (QCNN)
- ...

Intermediate measurements in the lab

Trapped Ion Quantum Information lab at U. Maryland ($\rightarrow$ Duke)

First demonstration of these protocols, in trapped ions! (arXiv:2112.05156)

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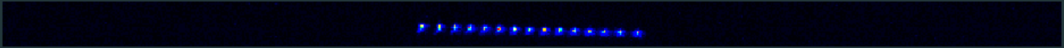

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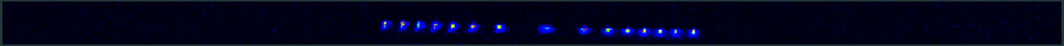

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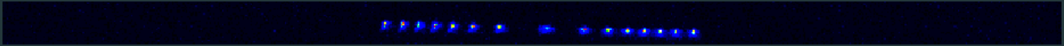

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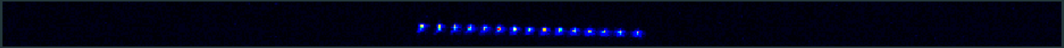

Experimental results for $f(x) = x^2 \bmod N$

Up and **right** is stronger evidence of quantumness

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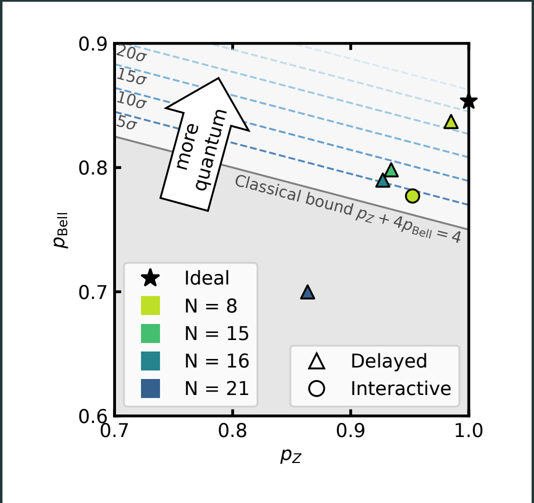

Looking forward

Bottleneck: Evaluating TCF on quantum superposition

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Improving the protocol itself:

- Remove trapdoor—symmetric key/hash-based cryptography [arXiv:2204.02063]
- Explore other protocols (verifiable sampling with good security?)

References + further reading

Numbers below are arXiv IDs; go to arxiv.org/abs/xxxx.xxxxx

Proofs of quantumness

- IQP sampling protocol [0809.0847]
- Breaking IQP protocol [1912.05547]
- First interactive proof based on trapdoor claw-free functions [1804.00640]
- Removing assumptions via random oracles [2005.04826]
- Removing assumptions via computational Bell test [2104.00687]
- Single-prover proofs from any multi-prover quantum game [2203.15877]
- Proofs using only random oracles [2204.02063]

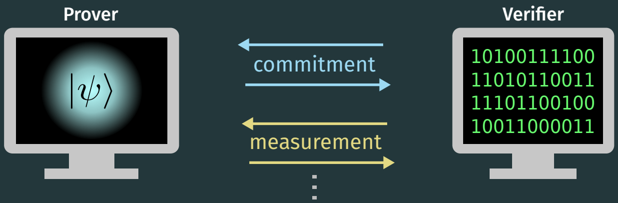
Other applications of quantum interactive proofs

- Certifiable quantum randomness [1804.00640]
- Remote state preparation [1904.06320]
- Verification of arbitrary quantum computations (!) [1804.01082]

Feel free to email me! Greg Kahanamoku-Meyer; gkm@berkeley.edu

Backup!

Hardness proof: rewinding



From a "proof of hardness" perspective:

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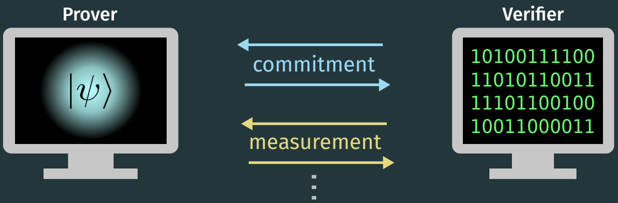

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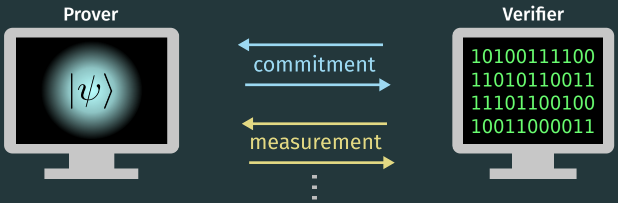

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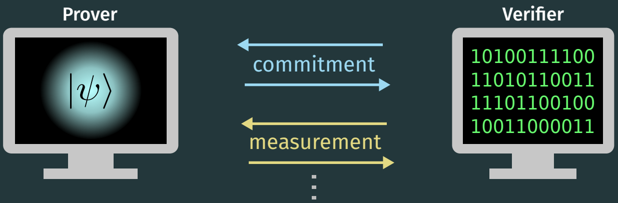

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"Rewinding" proof of hardness doesn't go through for quantum prover—can even use functions that are quantum claw-free!

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When we generate $\sum_x |x\rangle |f(x)\rangle$, **add redundancy to $f(x)$, for bit flip error detection!**

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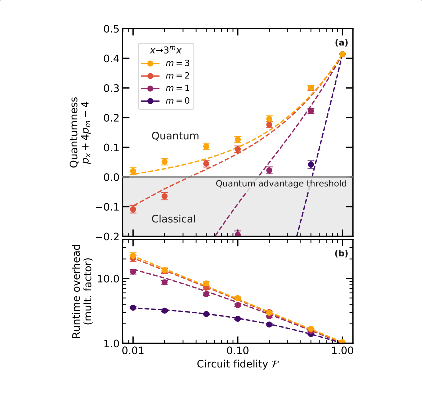

Numerical results for $x^2 \bmod N$ with $\log N = 512$ bits.

Here: make transformation $x^2 \bmod N \Rightarrow (kx)^2 \bmod k^2 N$

Improving circuit sizes

Most demanding step in all these protocols: evaluating TCF

$$\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$$

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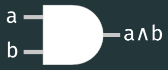
Protocol allows us to make circuits irreversible!

Technique: taking out the garbage

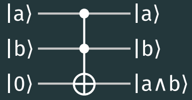
Goal: $\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$

When converting classical circuits to quantum:

Garbage bits: extra entangled outputs due to unitarity



Classical AND



Quantum AND (Toffoli)

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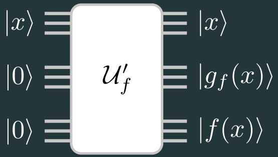

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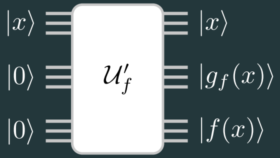

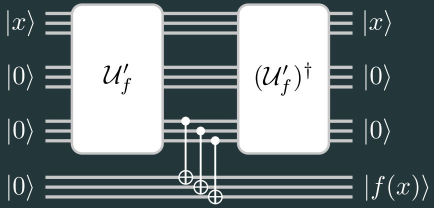

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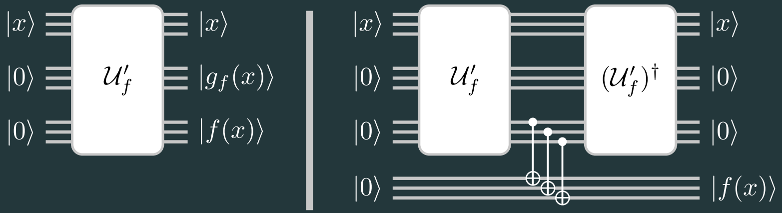

Lots of time and space overhead!

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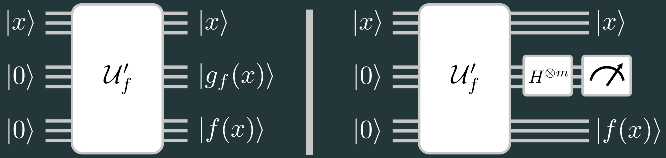
Let $\mathcal{U}_f'$ be a unitary generating garbage bits $g_f(x)$:



Can we "measure them away" instead?

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Measure garbage bits $g_f(x)$ in X basis, get some string h. End up with state:

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1024-bit $x^2 \bmod N$ in depth 10^5 (and can be improved?)

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Consider a matrix $P \in \{0,1\}^{k \times n}$ and "action" θ.

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Bremner, Jozsa, Shepherd '11: classically sampling worst-case IQP circuits would collapse polynomial heirarchy

Bremner, Montanaro, Shepherd '16: average case is likely hard as well

IQP proof of quantumness [Shepherd and Bremner, '08]

Let $\theta = \pi/8$, and s (secret) and P have the form:

$$P = \begin{bmatrix} \mathrm{G} \\ \hline \mathrm{R} \end{bmatrix}$$

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QR code: codewords have $\mathrm{wt}(c) \bmod 4 \in \{0, -1\}$

IQP proof of quantumness [Shepherd and Bremner, '08]

Let $\theta = \pi/8$, and s (secret) and P have the form:

$$P = \left[\begin{array}{c} \text{G} \\ \hline \text{R} \end{array}\right] \qquad \boldsymbol{Ps} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$G^\intercal$ is generator of Quadratic Residue code, R random.

$$\Pr[X^\intercal \cdot \boldsymbol{s} = 0] = \cos^2\left(\frac{\pi}{8}\right) \approx 0.85$$

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IQP: Hiding s

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Best classical: $\Pr[Y^\intercal \cdot s = 0] = ?$

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Scrambling preserves quantum success rate.

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Scrambling preserves quantum success rate.

Conjecture [SB '08]: Scrambling P cryptographically hides G (and equivalently $\mathbf{s}$)

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Assuming s hidden, can classical do better than 0.5? Try to take advantage properties of embedded code.

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QR code codewords are 50% even parity, 50% odd parity.

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Then:

$$y \cdot s = (G\boldsymbol{d}) \cdot (G\boldsymbol{e}) \pmod 2$$

Fact: $(G\boldsymbol{d}) \cdot (G\boldsymbol{e}) = 1$ iff $G\boldsymbol{d}$, $G\boldsymbol{e}$ both have odd parity.

IQP: Classical strategy [SB '08]

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Classical: $\Pr[Y^\intercal \cdot s = 0] = 0.75$

Consider choosing random $d, e \xleftarrow{\$} \{0,1\}^n$, and letting

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Then:

$$y \cdot s = (Gd) \cdot (Ge) \pmod 2$$

Fact: $(Gd) \cdot (Ge) = 1$ iff Gd, Ge both have odd parity.

Thus $y \cdot s = 0$ with probability $3/4$!

IQP: Can we do better classically? [GDKM ’19 arXiv:1912.05547]

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Can solve for $\mathbf{s}$! ... If M has high rank. Empirically it does!

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- Could pick a different G for which this attack would not succeed?
- Ultimately, would like to rely on standard cryptographic assumptions…

Quantum circuits for $x^2 \bmod N$

Goal: $\mathcal{U}\,|x\rangle\,|0\rangle = |x\rangle\,|x^2 \bmod N\rangle$

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Decompose this as

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with

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Advantages:

- Everything is diagonal (it's just a phase)!
- Modulo is automatic in the phase
- Simple decomposition into few-qubit gates

New goal: $\tilde{\mathcal{U}} |x\rangle |z\rangle = \exp\left(2\pi i \frac{x^2}{N} z\right) |x\rangle |z\rangle$

Decompose using "grade school" integer multiplication:

$$a \cdot b = \sum_{i,j} 2^{i+j} a_i b_j$$

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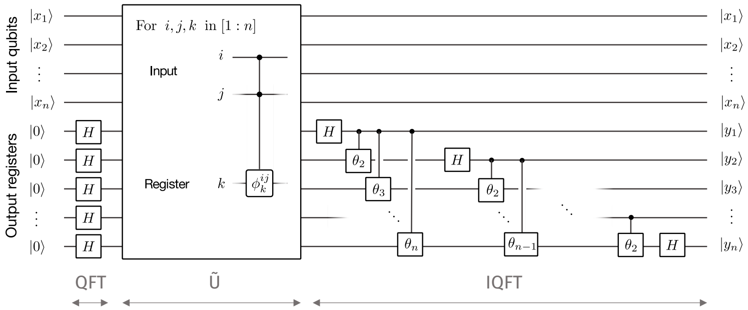
- Binary multiplication is AND
- "Apply phase whenever $x_i = x_j = z_k = 1$"
- These are CCPhase gates (of arb. phase)!

Leveraging the Rydberg blockade



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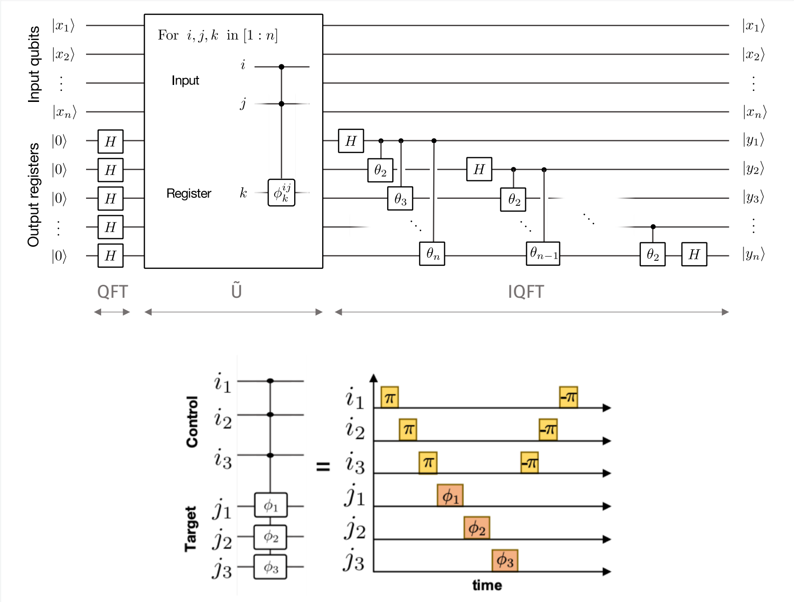

Decisional Diffie-Hellman (DDH)

Problem (not TCF): Consider a group $\mathbb{G}$ of order N, with generator g.
Given the tuple (g, g^a, g^b, g^c), determine if $c = ab$.

Elliptic curve crypto.: $\log N \sim 160$ bits is as hard as 1024 bit factoring!!

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How to build a TCF?

Trapdoor [Peikert, Waters '08; Freeman et al. '10]: linear algebra in the exponent

Claw-free [GDKM et al. '21 (arXiv:2104.00687)]: collisions in linear algebra in the exponent!

Full protocol

Prover (quantum)

Verifier (classical)

Round 1

Verifier: 1. Sample $(f_i, t) \leftarrow \mathsf{Gen}(1^n)$

$\xleftarrow{f_i}$

Prover: 2. Generate state $\sum_{x=0}^{N/2} |x\rangle_{\mathsf{x}} |f_i(x)\rangle_{\mathsf{y}}$

Prover: 3. Measure y register, yielding bitstring y
State is now $(|x_0\rangle + |x_1\rangle)_{\mathsf{x}} |y\rangle_{\mathsf{y}}$;
y register can be discarded

$\xrightarrow{y}$

Verifier: 4. Using trapdoor t compute x_0 and x_1

Verifier: 5. Randomly choose to request a preimage or continue

$\xleftarrow{\text{choice}}$

If preimage requested:

Prover: 6a. Projectively measure x register, yielding x

$\xrightarrow{x}$

Verifier: 7a. If $x \in \{x_0, x_1\}$ return Accept

Otherwise, continue:

Round 2

Verifier: 6b. Choose random bitstring r

$\xleftarrow{r}$

Prover: 7b. Add one ancilla b; use CNOTs to compute
$|r \cdot x_0\rangle_{\mathsf{b}} |x_0\rangle_{\mathsf{x}} + |r \cdot x_1\rangle_{\mathsf{b}} |x_1\rangle_{\mathsf{x}}$ where
$r \cdot x$ is bitwise inner product

Prover: 8b. Measure x register in Hadamard basis, yielding a string d. Discard x, state is now
$|\psi\rangle_{\mathsf{b}} \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$

$\xrightarrow{d}$

Verifier: 9b. Using r, x_0, x_1, d, determine $|\psi\rangle_{\mathsf{b}}$

Round 3

Verifier: 10b. Choose random $m \in \{\frac{\pi}{4}, -\frac{\pi}{4}\}$

$\xleftarrow{m}$

Prover: 11b. Measure ancilla b in the rotated basis

$$\left\{ \begin{matrix} \cos\left(\frac{m}{2}\right)|0\rangle + \sin\left(\frac{m}{2}\right)|1\rangle \\ \cos\left(\frac{m}{2}\right)|1\rangle - \sin\left(\frac{m}{2}\right)|0\rangle \end{matrix} \right\},$$ yielding a bit b

$\xrightarrow{b}$

Verifier: 11b. If b was likely given $|\psi\rangle_{\mathsf{b}}$ return Accept