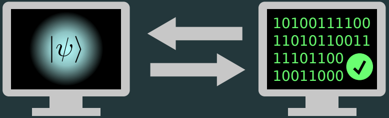
Classical verification of quantum computational advantage



Gregory D. Kahanamoku-Meyer

March 15, 2022

arXiv:2104.00687 (theory)
arXiv:2112.05156 (expt.)

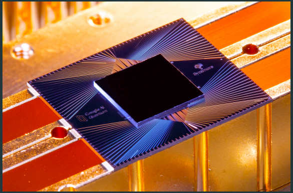
Theory collaborators:

- Norman Yao (Berkeley → Harvard)
- Umesh Vazirani (Berkeley)
- Soonwon Choi (Berkeley → MIT)

Berkeley
UNIVERSITY OF CALIFORNIA

Quantum computational advantage

Recent first experimental demonstrations:



Random circuit sampling
[Arute et al., Nature '19]

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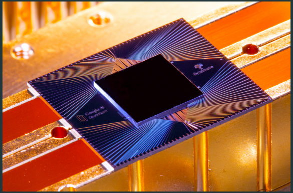

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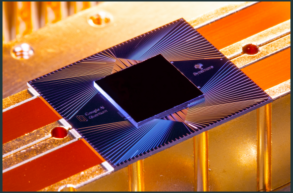

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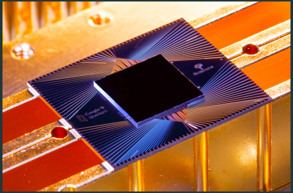

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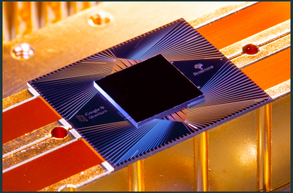

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Quantum is the only reasonable explanation for observed behavior,
under some assumptions about the inner workings of the device

“Black-box” quantum computational advantage

Stronger: rule out all classical hypotheses, even pathological!

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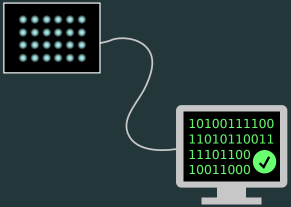

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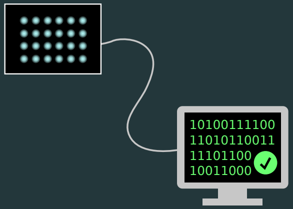

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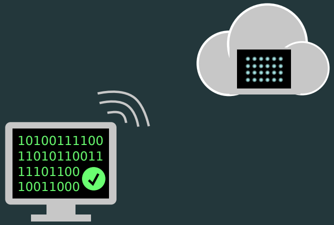

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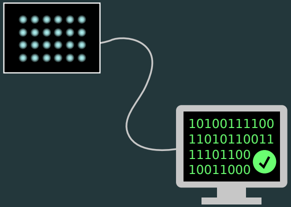

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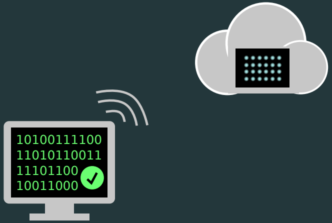

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Reframing: disprove null hypothesis that output was generated classically.

Noisy intermediate scale verifiable quantum advantage

Trivial solution: Shor's algorithm

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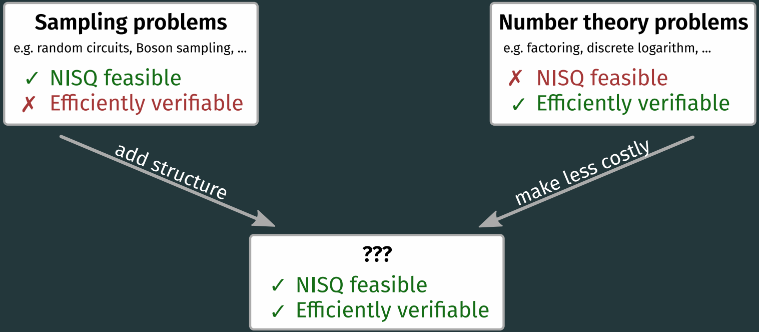
NISQ: Noisy Intermediate-Scale Quantum devices



Sampling problems

e.g. random circuits, Boson sampling, ...

✓ NISQ feasible
✗ Efficiently verifiable

Number theory problems

e.g. factoring, discrete logarithm, ...

✗ NISQ feasible
✓ Efficiently verifiable

add structure

make less costly

???

✓ NISQ feasible
✓ Efficiently verifiable

Making number theoretic problems less costly

Fully solving a problem like factoring is “overkill”

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Can we demonstrate quantum *capability* without needing to solve such a hard problem?

Zero-knowledge proofs: differentiating colors

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How can they use a red ball and green ball to convince you that they see color?

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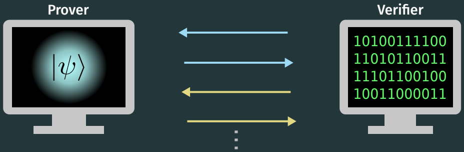
Seeing color ⇔ Quantum capability

Goal: find protocol **as verifiable and classically hard as factoring**—
but **less expensive than actually finding factors (via Shor)**

Interactive proofs of quantumness

Multiple rounds of interaction between the prover and verifier



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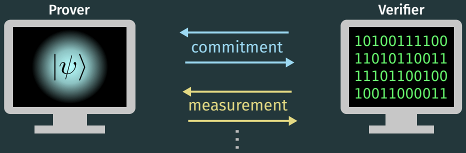

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Round 2: Verifier asks for measurement in specific basis, prover performs it

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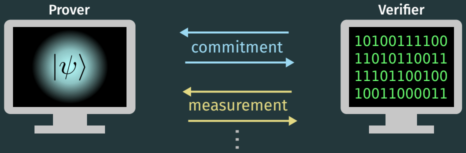

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> By randomizing choice of basis and repeating interaction,
> can ensure prover would respond correctly in *any* basis

Brakerski, Christiano, Mahadev, Vidick, Vazirani '18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

State commitment (round 1): trapdoor claw-free functions

How does the prover commit to a state?

Consider a **2-to-1** function f:
for all y in range of f, there exist (x_0, x_1) such that $y = f(x_0) = f(x_1)$.

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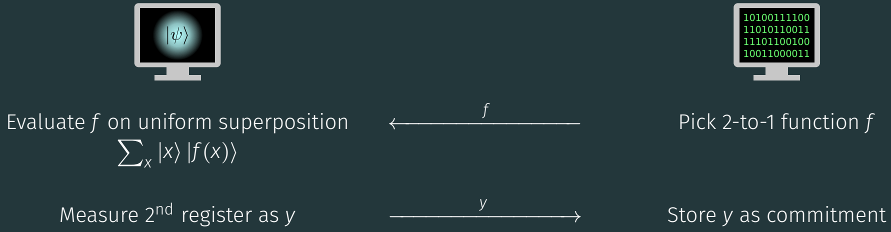

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Generating a valid state without trapdoor uses superposition + wavefunction collapse—inherently quantum!

Trapdoor claw-free function example

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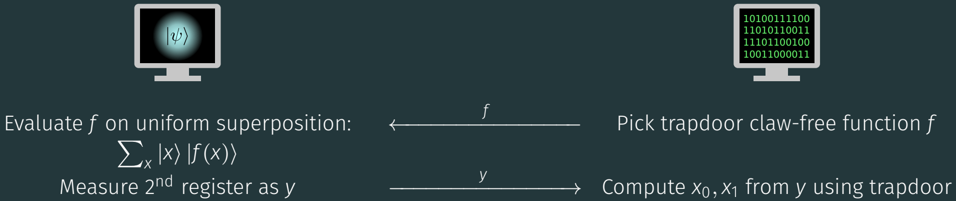
Example: $4^2 \equiv 11^2 \equiv 16 \pmod{35}$; and $11 - 4 = 7$

Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18



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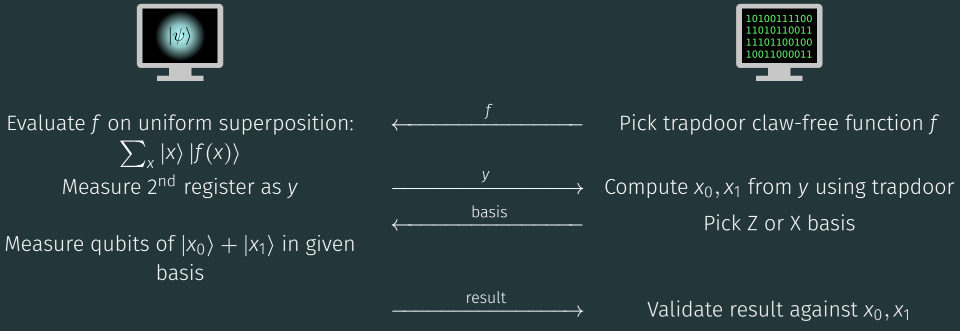

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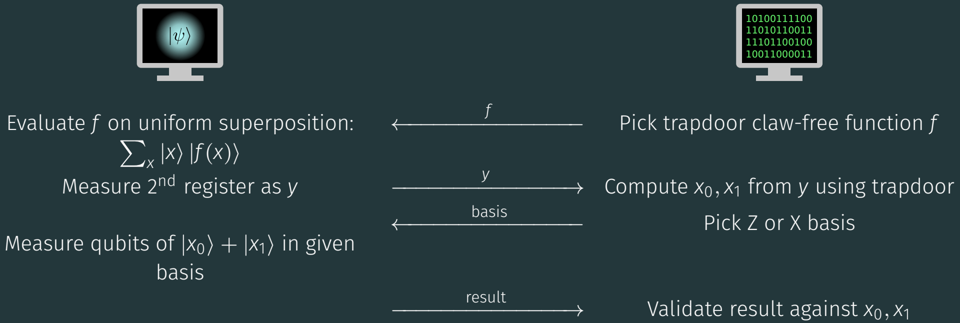

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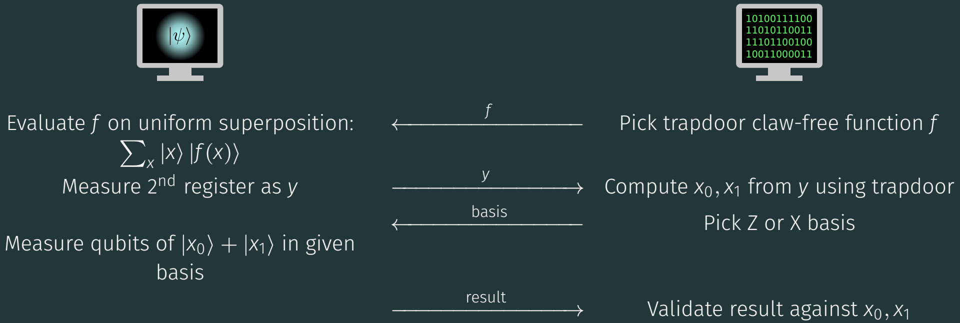

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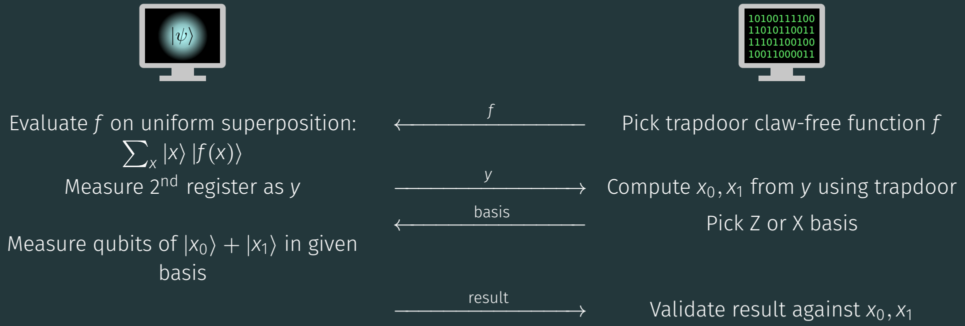

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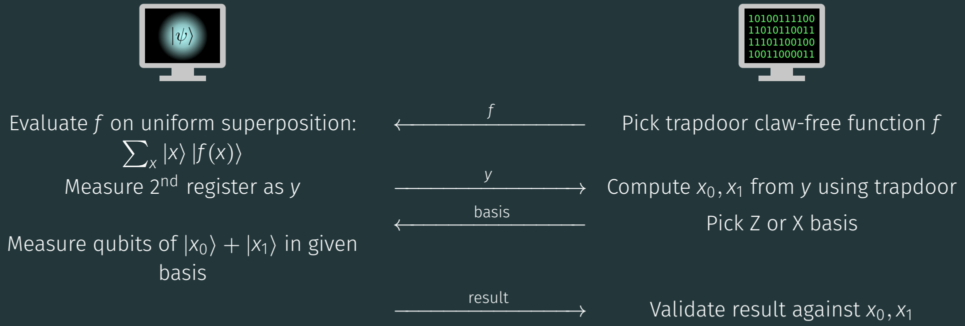

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Protocol requires strong claw-free property:

For any x_0, hard to find even **a single bit** about x_1.

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Trapdoor claw-free functions

Function family	Trapdoor	Claw-free	Strong claw-free
Learning-with-Errors [1]	✓	✓	✓
Ring Learning-with-Errors [2]	✓	✓	✗
$x^2 \bmod N$ [3]	✓	✓	✗
Diffie-Hellman [3]	✓	✓	✗

[1] Brakerski, Christiano, Mahadev, Vidick, Vazirani '18 (arXiv:1804.00640)

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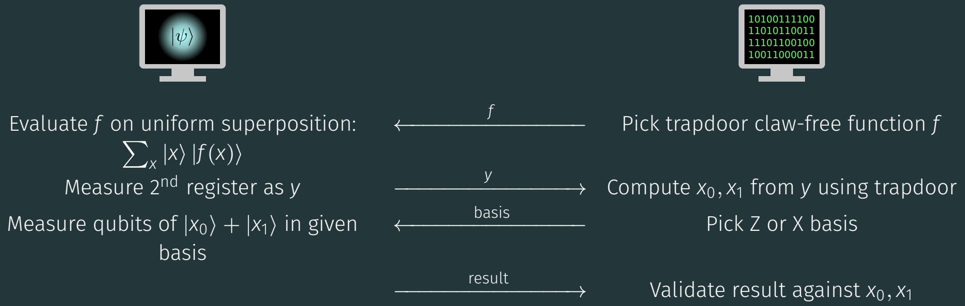

Evaluate f on uniform superposition: $\sum_x |x\rangle |f(x)\rangle$ $\xleftarrow{\quad f \quad}$ Pick trapdoor claw-free function f

Measure 2nd register as y $\xrightarrow{\quad y \quad}$ Compute x_0, x_1 from y using trapdoor

Measure qubits of $|x_0\rangle + |x_1\rangle$ in given basis $\xleftarrow{\text{basis}}$ Pick Z or X basis

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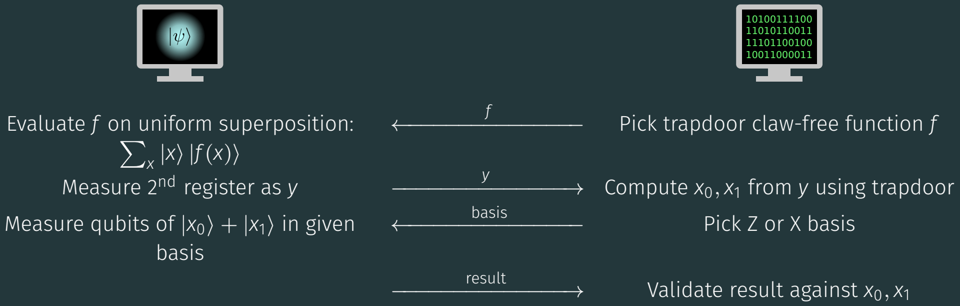

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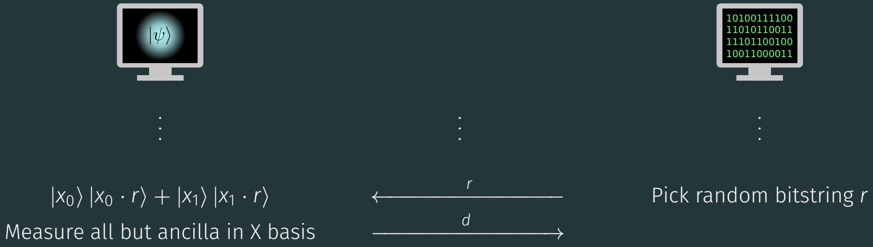
Replace X basis measurement with “single-qubit Bell test”

Interactive measurement: computational Bell test

Two-step process: “condense” x_0, x_1 into a single qubit, and then do a “Bell test.”



$|x_0\rangle |x_0 \cdot r\rangle + |x_1\rangle |x_1 \cdot r\rangle$

Measure all but ancilla in X basis

Pick random bitstring r

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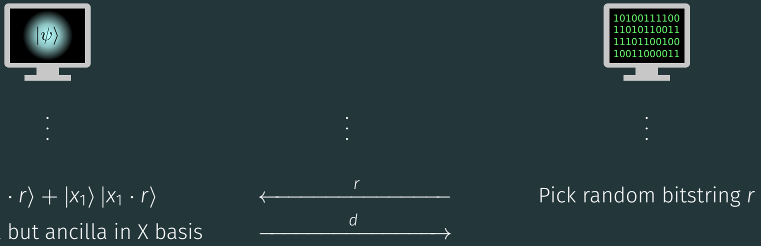

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Now 1-qubit state: $|0\rangle$ or $|1\rangle$ if $x_0 \cdot r = x_1 \cdot r$, otherwise $|+\rangle$ or $|-\rangle$.

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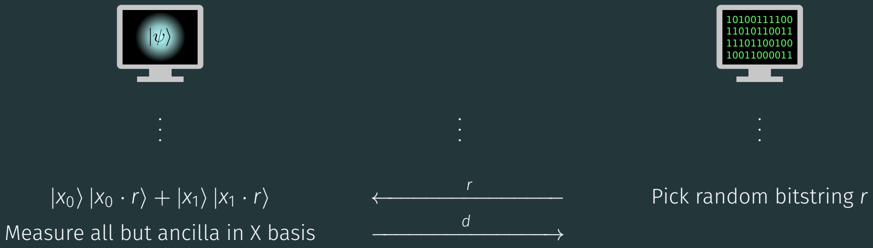

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Now 1-qubit state: $|0\rangle$ or $|1\rangle$ if $x_0 \cdot r = x_1 \cdot r$, otherwise $|+\rangle$ or $|-\rangle$. Polarization hidden via:

Cryptographic secret (here) $\Leftrightarrow$ Non-communication (Bell test)

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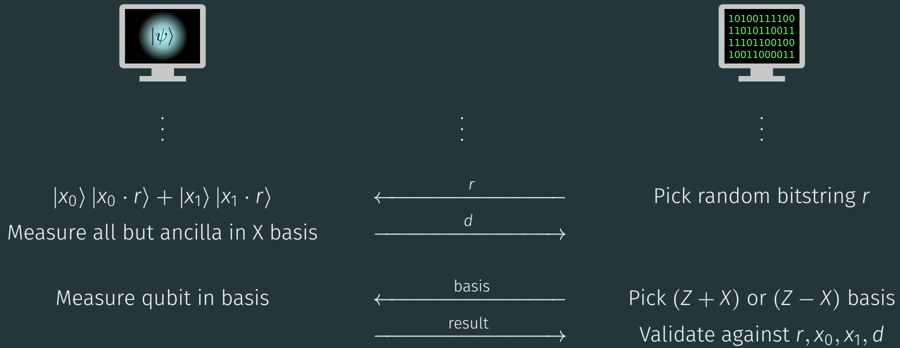

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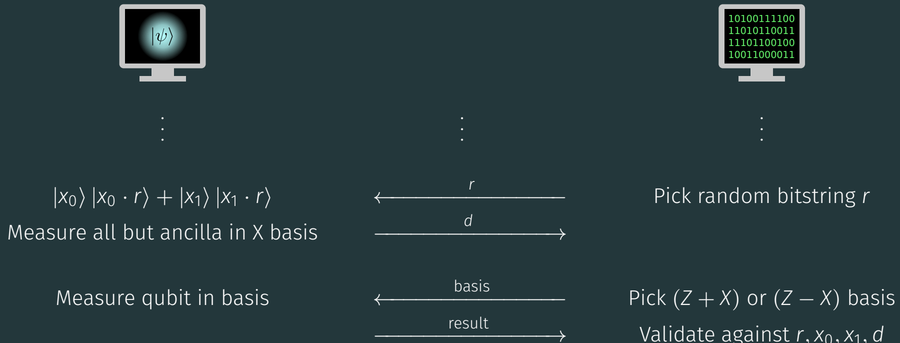

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This protocol can use any trapdoor claw-free function!

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Computational Bell test: classical bound

Run protocol many times, collect statistics.

p_Z: Success rate for Z basis measurement.

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Note: Let $p_Z = 1$. Then for p_{Bell}:

Classical bound 75%, ideal quantum $\sim$ 85%. Same as regular Bell test!

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Asymptotically: evaluating $x^2 \bmod N$ requires $\mathcal{O}(n \log n)$ gates;
$a^x \bmod N$ in Shor requires $\mathcal{O}(n^2 \log n)$

(can also use other TCFs, and other optimizations...)

Moving towards efficiently-verifiable quantum advantage in the near term

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- Measurement-based uncomputation scheme [2]

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Intermediate measurements in the lab

Trapped Ion Quantum Information lab at U. Maryland (→ Duke)

First demonstration of these protocols, in trapped ions! (arXiv:2112.05156)

Dr. Daiwei Zhu

Prof. Crystal Noel

Prof. Christopher Monroe

and others!

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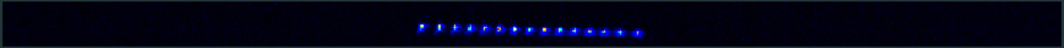

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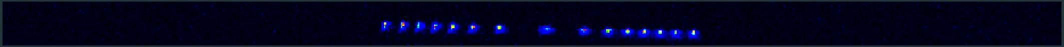

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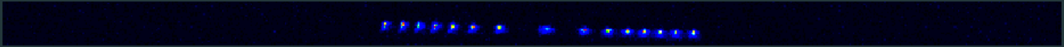

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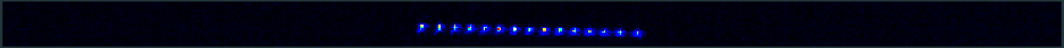
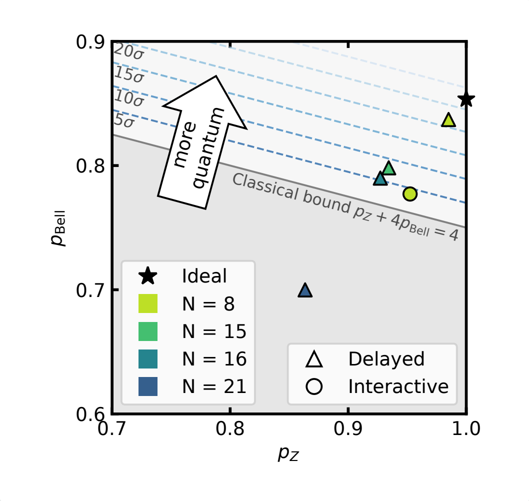


Interactive proofs on a few qubits

Experimental results for $f(x) = x^2 \bmod N$

Up and **right** is stronger evidence of quantumness

GDKM, D. Zhu, et al. (arXiv:2112.05156)



Looking forward

Bottleneck: Evaluating TCF on quantum superposition

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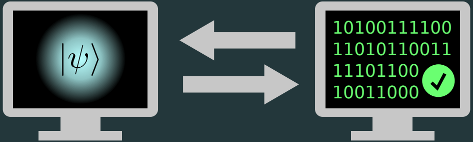
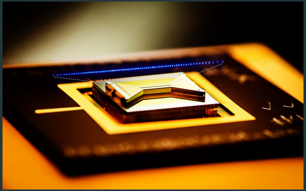
- Explore other protocols (verifiable sampling?)

Questions?

arXiv:2104.00687 (theory)



arXiv:2112.05156 (experiment)



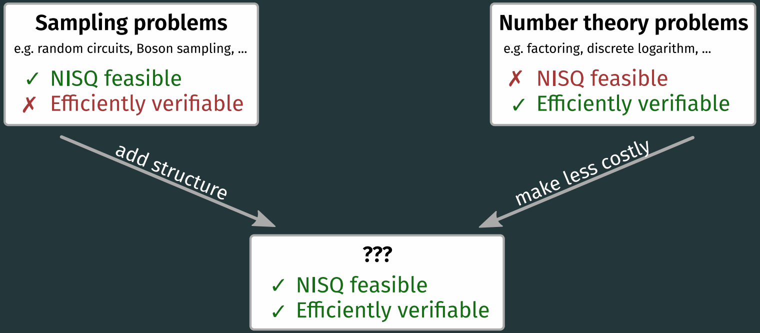
Gregory D. Kahanamoku-Meyer

gregdmeyer.github.io

Backup!

Noisy intermediate scale verifiable quantum advantage

NISQ: Noisy Intermediate-Scale Quantum devices



Sampling problems

e.g. random circuits, Boson sampling, ...

✓ NISQ feasible

✗ Efficiently verifiable

Number theory problems

e.g. factoring, discrete logarithm, ...

✗ NISQ feasible

✓ Efficiently verifiable

add structure

make less costly

???

✓ NISQ feasible

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Adding structure to sampling problems

Generically: seems hard.

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Example: sampling "IQP" circuits (products of Pauli X's)

$$H = X_0 X_1 X_3 + X_1 X_2 X_4 X_5 + \cdots \tag{1}$$

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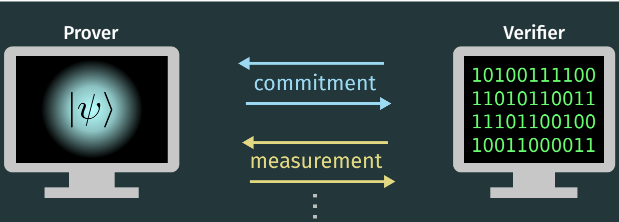
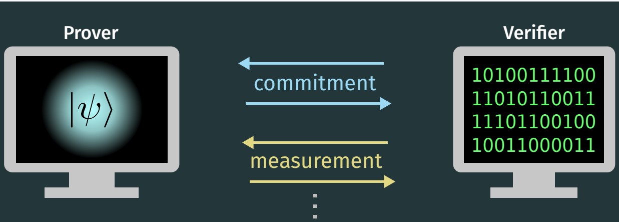
Adding structure opens opportunities for classical cheating

Hardness proof: rewinding



From a “proof of hardness” perspective:

Hardness proof: rewinding



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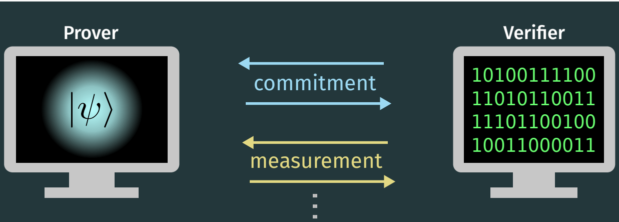

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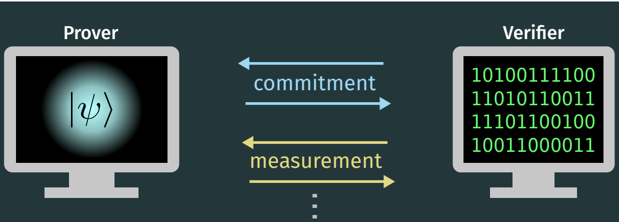

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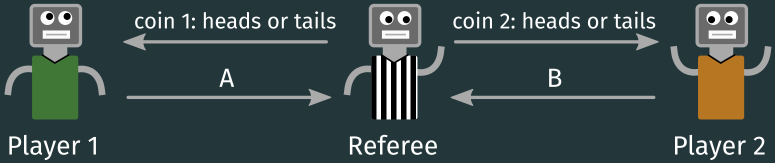
“Rewinding” proof of hardness doesn’t go through for quantum prover—can even use functions that are quantum claw-free!

The CHSH game (Bell test)

Cooperative two-player game; players can't communicate (non-local).



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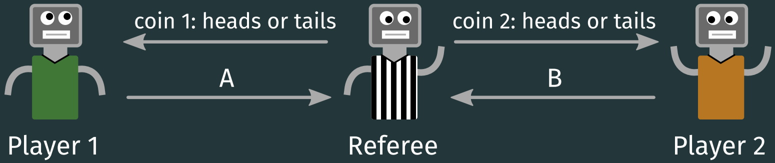

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Classical optimal strategy: return equal values, hope you didn’t both get heads. 75% success rate.

Can we do better with entanglement?

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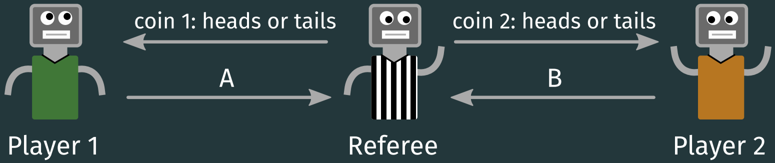

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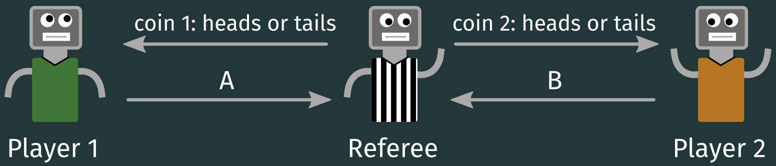
Consider the Bell pair: $|\psi\rangle = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$

The CHSH game (Bell test)



coin 1: heads or tails

A

coin 2: heads or tails

B

Player 1

Referee

Player 2

If anyone receives tails, want $A = B$. If both get heads, want $A \neq B$.

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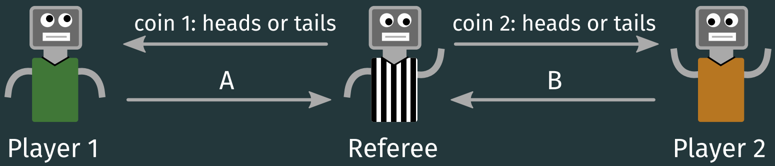

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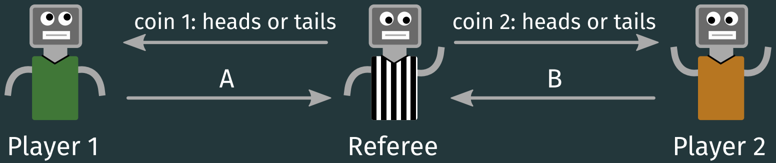

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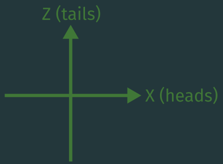

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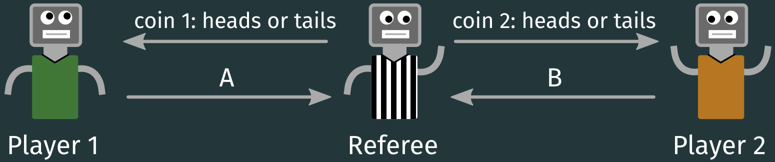

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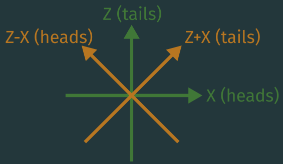


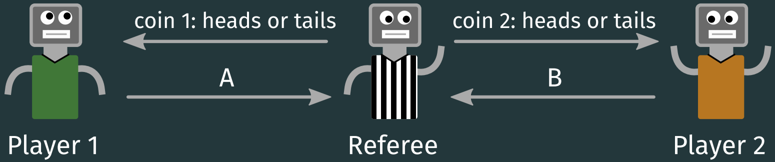
Z (tails)

Z-X (heads)

Z+X (tails)

X (heads)

The CHSH game (Bell test)



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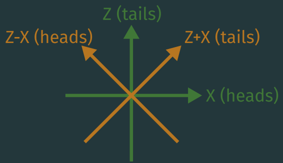

Quantum: $\cos^2(\pi/8) \approx 85\%$

Classical: 75%

Technique: postselection

How to deal with high fidelity requirement? Naively need $\sim 83\%$ overall circuit fidelity to pass.

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When we generate $\sum_x |x\rangle\,|f(x)\rangle$, **add redundancy to $f(x)$, for bit flip error detection!**

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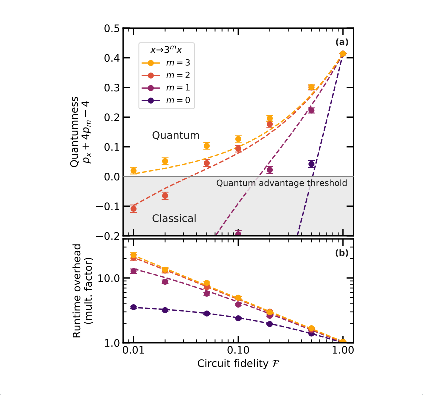

Numerical results for $x^2 \bmod N$ with $\log N = 512$ bits.

Here: make transformation $x^2 \bmod N \Rightarrow (kx)^2 \bmod k^2 N$

Improving circuit sizes

Most demanding step in all these protocols: evaluating TCF

$$\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$$

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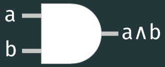
Protocol allows us to make circuits irreversible!

Technique: taking out the garbage

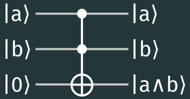
Goal: $\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$

When converting classical circuits to quantum:

Garbage bits: extra entangled outputs due to unitarity



Classical AND



Quantum AND (Toffoli)

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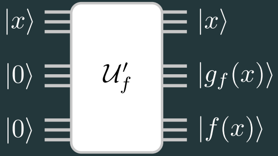

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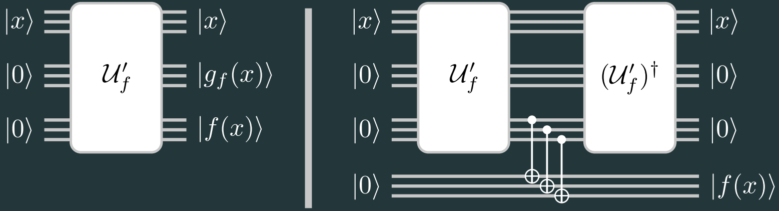

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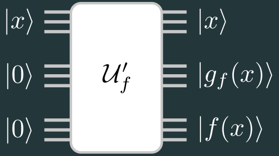

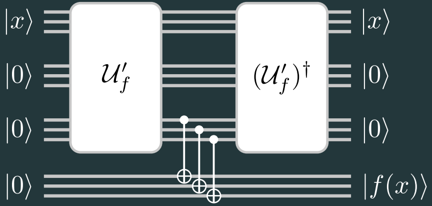

Lots of time and space overhead!

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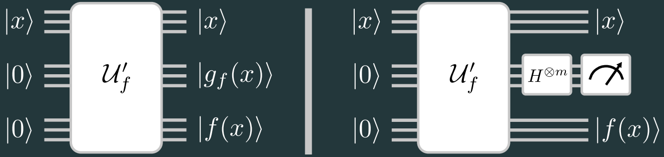

Can we “measure them away” instead?

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Measure garbage bits $g_f(x)$ in X basis, get some string h. End up with state:

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1024-bit $x^2 \bmod N$ in depth 10^5 (and can be improved?)

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Decompose this as

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Advantages:

- Everything is diagonal (it’s just a phase)!
- Modulo is automatic in the phase
- Simple decomposition into few-qubit gates

Implementation

New goal: $\tilde{\mathcal{U}} \left|x\right\rangle \left|z\right\rangle = \exp\left(2\pi i \frac{x^2}{N} z\right) \left|x\right\rangle \left|z\right\rangle$

Decompose using “grade school” integer multiplication:

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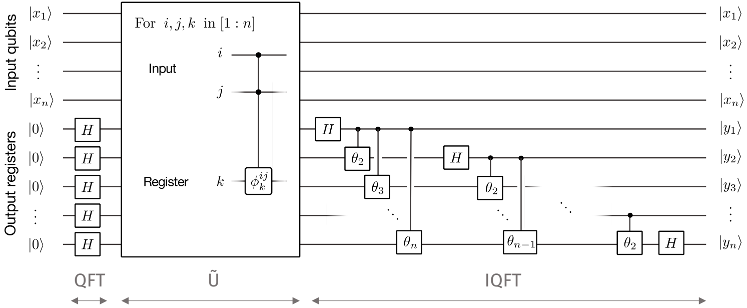
- Binary multiplication is AND
- “Apply phase whenever $x_i = x_j = z_k = 1$”
- These are CCPhase gates (of arb. phase)!

Leveraging the Rydberg blockade



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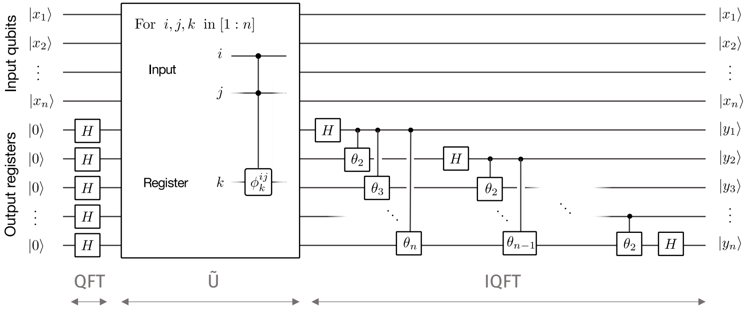

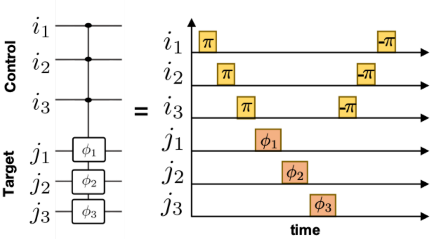

Decisional Diffie-Hellman (DDH)

Problem (not TCF): Consider a group $\mathbb{G}$ of order N, with generator g. Given the tuple (g, g^a, g^b, g^c), determine if $c = ab$.

Elliptic curve crypto.: $\log N \sim 160$ bits is as hard as 1024 bit factoring!!

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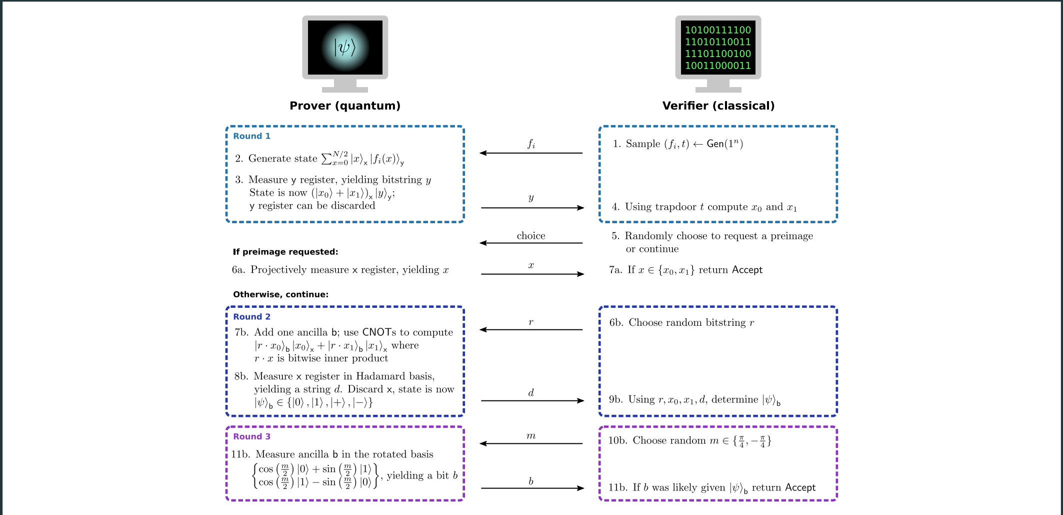
How to build a TCF?

Trapdoor [Peikert, Waters ’08; Freeman et al. ’10]: linear algebra in the exponent

Claw-free [GDKM et al. ’21 (arXiv:2104.00687)]: collisions in linear algebra in the exponent!

Full protocol



Prover (quantum)

Verifier (classical)

Round 1

1. Sample $(f_i, t) \leftarrow \mathsf{Gen}(1^n)$

$\xleftarrow{f_i}$

2. Generate state $\sum_{x=0}^{N/2} |x\rangle_{\mathsf{x}} |f_i(x)\rangle_{\mathsf{y}}$

3. Measure y register, yielding bitstring y
State is now $(|x_0\rangle + |x_1\rangle)_{\mathsf{x}} |y\rangle_{\mathsf{y}}$;
y register can be discarded

$\xrightarrow{y}$

4. Using trapdoor t compute x_0 and x_1

5. Randomly choose to request a preimage or continue

$\xleftarrow{\text{choice}}$

If preimage requested:

6a. Projectively measure x register, yielding x

$\xrightarrow{x}$

7a. If $x \in \{x_0, x_1\}$ return Accept

Otherwise, continue:

Round 2

6b. Choose random bitstring r

$\xleftarrow{r}$

7b. Add one ancilla b; use CNOTs to compute
$|r \cdot x_0\rangle_{\mathsf{b}} |x_0\rangle_{\mathsf{x}} + |r \cdot x_1\rangle_{\mathsf{b}} |x_1\rangle_{\mathsf{x}}$ where
$r \cdot x$ is bitwise inner product

8b. Measure x register in Hadamard basis, yielding a string d. Discard x, state is now
$|\psi\rangle_{\mathsf{b}} \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$

$\xrightarrow{d}$

9b. Using r, x_0, x_1, d, determine $|\psi\rangle_{\mathsf{b}}$

Round 3

10b. Choose random $m \in \{\frac{\pi}{4}, -\frac{\pi}{4}\}$

$\xleftarrow{m}$

11b. Measure ancilla b in the rotated basis

$$\left\{ \begin{matrix} \cos\left(\frac{m}{2}\right)|0\rangle + \sin\left(\frac{m}{2}\right)|1\rangle \\ \cos\left(\frac{m}{2}\right)|1\rangle - \sin\left(\frac{m}{2}\right)|0\rangle \end{matrix} \right\},$$ yielding a bit b

$\xrightarrow{b}$

11b. If b was likely given $|\psi\rangle_{\mathsf{b}}$ return Accept