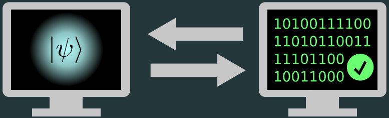


Classical verification of quantum computational advantage

Gregory D. Kahanamoku-Meyer

February 9, 2022

arXiv:2104.00687 (theory)
arXiv:2112.05156 (expt.)

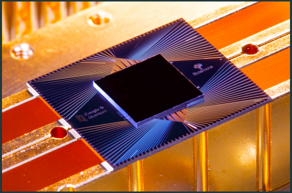
QUANTUM SYSTEMS ACCELERATOR
Catalyzing the Quantum Ecosystem

Berkeley
UNIVERSITY OF CALIFORNIA

BERKELEY LAB

Quantum computational advantage

Recent experimental demonstrations:



Random circuit sampling
[Arute et al., Nature '19]

Gaussian boson sampling
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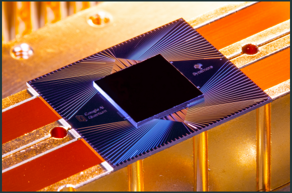

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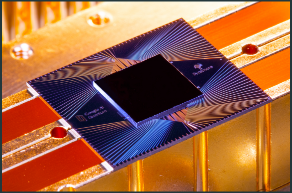

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"... [Rule] out alternative [classical] hypotheses that might be plausible in this experiment" [Zhong et al.]

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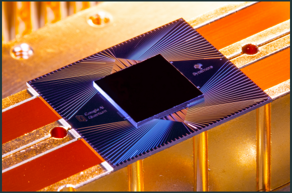

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Quantum is the only reasonable explanation for observed behavior

“Black-box” quantum computational advantage

Stronger: rule out all classical hypotheses, even pathological!

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Explicitly perform a “proof of quantumness”

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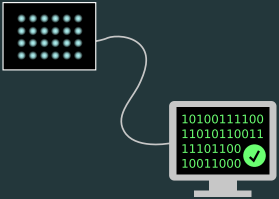

Local: rigorously refute
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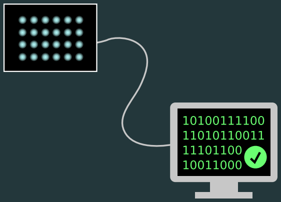

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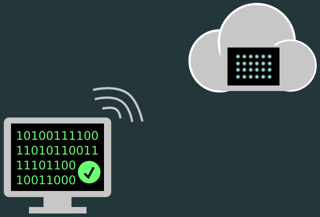
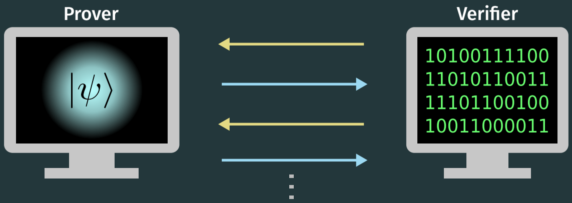


Remote: validate an untrusted quantum cloud service

Interactive proofs

Multiple rounds of interaction between the prover and verifier



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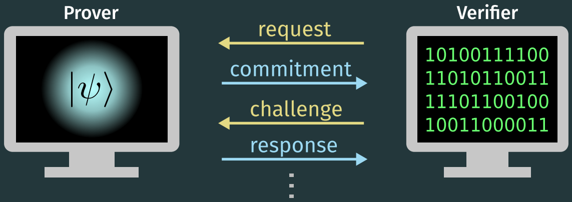

Prover must commit data before learning the challenge

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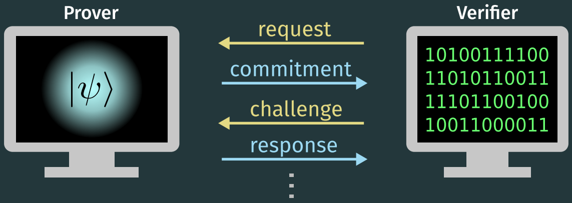
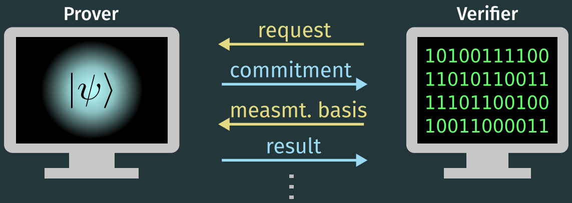


Prover must commit data before learning the challenge

Via repetition can establish that prover can respond correctly to *any* challenge.

Interactive proofs of quantumness



Round 1: Prover commits to a specific quantum state

Round 2: Verifier asks for measurement in specific basis

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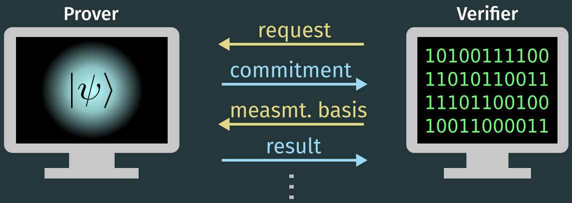

Round 1: Prover commits to a specific quantum state

Round 2: Verifier asks for measurement in specific basis

> By randomizing choice of basis and repeating interaction, can ensure prover would respond correctly in *any* basis

Brakerski, Christiano, Mahadev, Vazirani, Vidick '18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

State commitment (round 1): trapdoor claw-free functions

How does the prover commit to a state?

Consider a **2-to-1** function f:
for all y in range of f, there exist (x_0, x_1) such that $y = f(x_0) = f(x_1)$.

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Evaluate f on uniform superposition $\sum_x |x\rangle\,|f(x)\rangle$ $\xleftarrow{\quad f \quad}$ Pick 2-to-1 function f

Measure 2$^{\text{nd}}$ register as y $\xrightarrow{\quad y \quad}$ Store y as commitment

Prover has committed to the state $(|x_0\rangle + |x_1\rangle)\,|y\rangle$

State commitment (round 1): trapdoor claw-free functions

Prover has committed to $(|x_0\rangle + |x_1\rangle)\,|y\rangle$ with $y = f(x_0) = f(x_1)$

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The only path to a valid state without trapdoor is by superposition + wavefunction collapse—inherently quantum!

[BCMVV ’18] protocol

Prover

Verifier

Pick trapdoor claw-free function f $\xleftarrow{f}$ Evaluate f on uniform superposition: $\sum_x |x\rangle |f(x)\rangle$

Measure 2nd register as y $\xrightarrow{y}$ Compute x_0, x_1 from y using trapdoor

Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18 (arXiv:1804.00640)

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Measure qubits of $|x_0\rangle + |x_1\rangle$ in given basis		
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Perform experiment many times,
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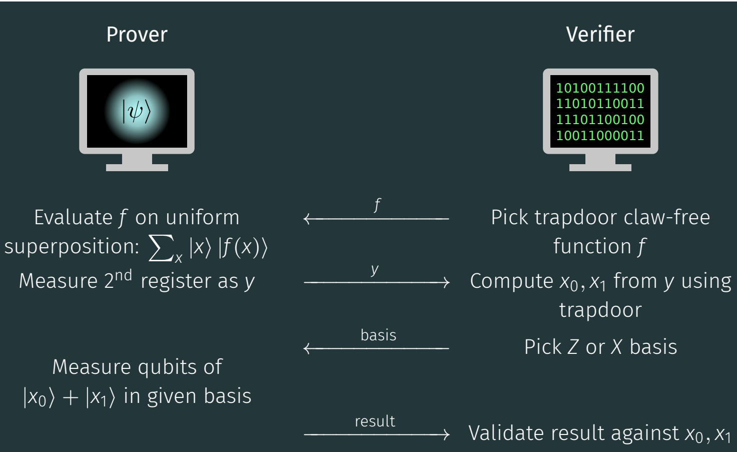

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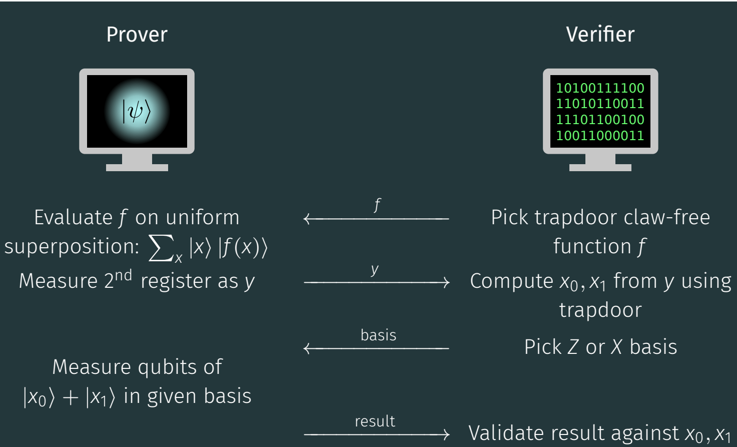
Validate result against x_0, x_1

Classical bound: $p_Z + 2p_X < 2 + \epsilon$

Ideal quantum: $p_Z + 2p_X = 3$

[BCMVV ’18] protocol



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Subtlety: claw-free alone does *not* imply classical bound!
Learning-with-Errors TCF has adaptive hardcore bit

Trapdoor claw-free functions

TCF	Trapdoor	Claw-free	Adaptive hard-core bit
LWE [1]	✓	✓	✓
Ring-LWE [2]	✓	✓	✗
$x^2 \bmod N$ [3]	✓	✓	✗
Diffie-Hellman [3]	✓	✓	✗

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Interactive measurement: computational Bell test

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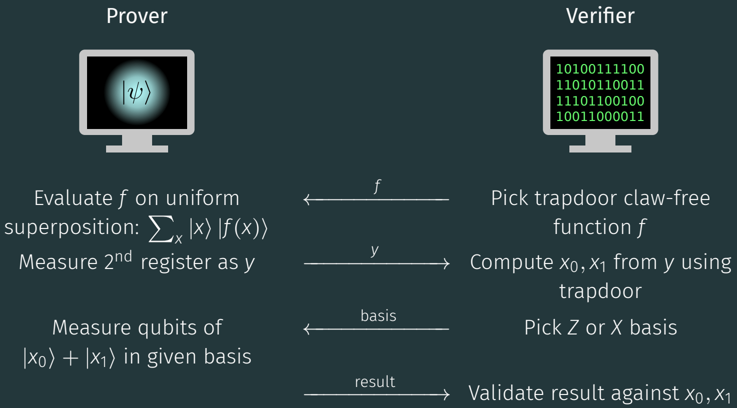

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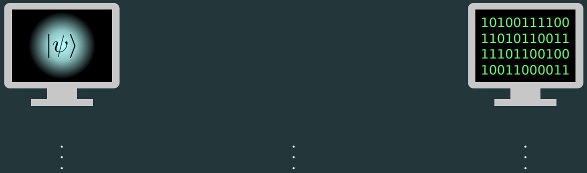
Replace X basis measurement with “1-player CHSH game.”

Interactive measurement: computational Bell test

Replace X basis measurement with two-step process:
“condense” x_0, x_1 into a single qubit, and then do a “Bell test.”



$|x_0\rangle |x_0 \cdot r\rangle + |x_1\rangle |x_1 \cdot r\rangle$ $\xleftarrow{\quad r \quad}$ Pick random bitstring r

Measure all but ancilla in X basis $\xrightarrow{\quad d \quad}$

GDKM, Choi, Vazirani, Yao ’21 (arXiv:2104.00687)

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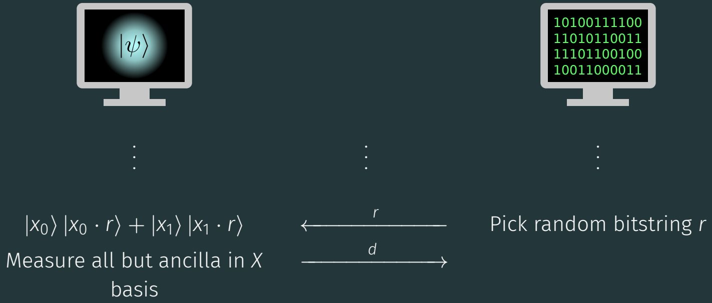

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Now single-qubit state: $|0\rangle$ or $|1\rangle$ if $x_0 \cdot r = x_1 \cdot r$, otherwise $|+\rangle$ or $|-\rangle$.

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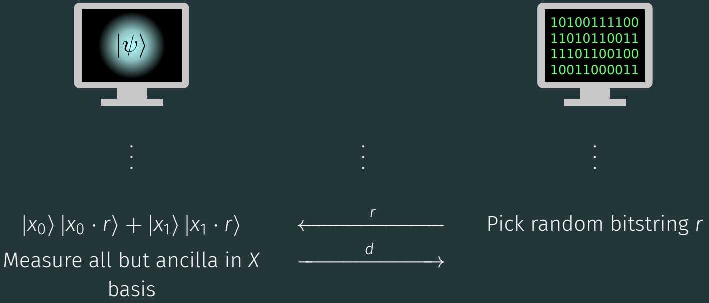

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Polarization hidden via:

Cryptographic secret (here) $\Leftrightarrow$ Non-communication (Bell test)

GDKM, Choi, Vazirani, Yao ’21 (arXiv:2104.00687)

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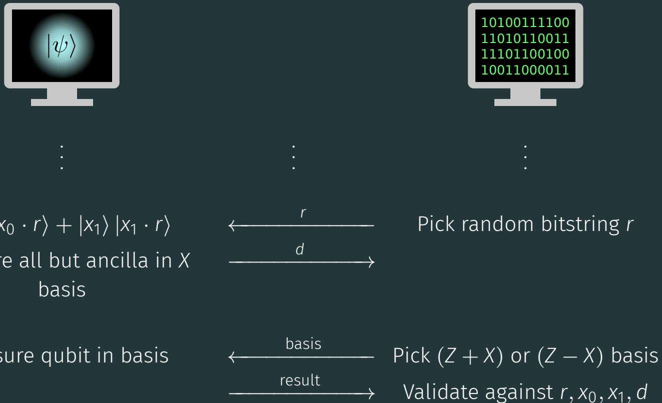

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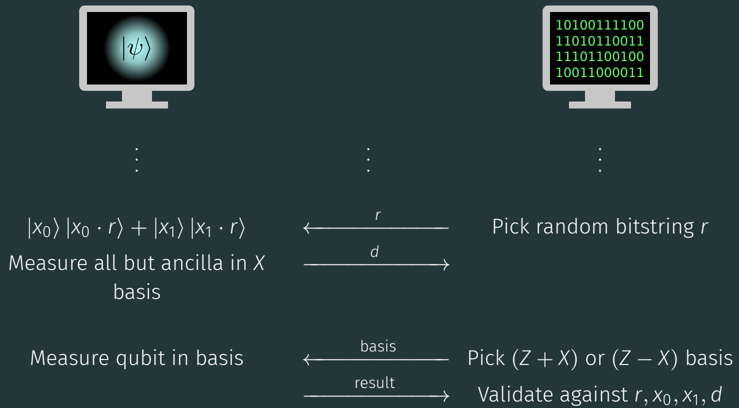

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Now can use any trapdoor claw-free function!

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

Computational Bell test: classical bound

Run protocol many times, collect statistics.

p_Z: Success rate for Z basis measurement.

p_{CHSH}: Success rate when performing CHSH-type measurement.

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Classical bound: $p_Z + 4p_{\text{CHSH}} - 4 < \epsilon$

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Note: Let $p_Z = 1$. Then for p_{CHSH}:

Classical bound 75%, ideal quantum $\sim$ 85%. Same as regular CHSH!

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

Moving towards full efficiently-verifiable quantum adv. on NISQ

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- Need to measure subsystem while maintaining coherence on other qubits

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- Removing need for adaptive hardcore bit allows “easier” TCFs

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Circuit sizes

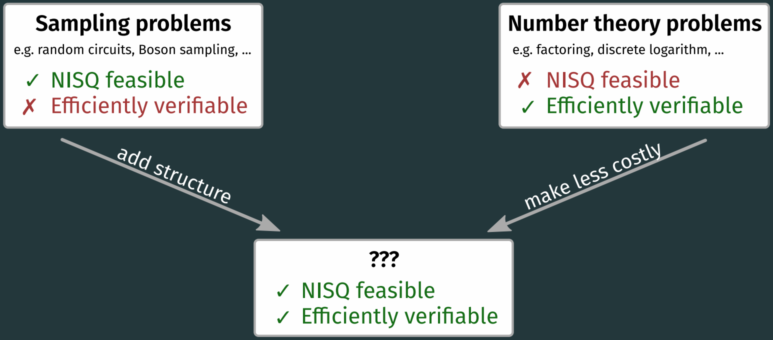
- Removing need for adaptive hardcore bit allows “easier” TCFs
- Measurement-based uncomputation scheme [arXiv:2104.00687]
- … hopefully can continue making theory improvements!

Backup

NISQ verifiable quantum advantage

Trivial solution: Shor's algorithm … but we want to do near-term!

NISQ: Noisy Intermediate-Scale Quantum devices



Sampling problems

e.g. random circuits, Boson sampling, …

✓ NISQ feasible

✗ Efficiently verifiable

Number theory problems

e.g. factoring, discrete logarithm, …

✗ NISQ feasible

✓ Efficiently verifiable

???

✓ NISQ feasible

✓ Efficiently verifiable